Cambridge
International
AS\&A Level

## Cambridge Assessment International Education

Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS
9709/61
Paper 6
May/June 2019
MARK SCHEME
Maximum Mark: 50

## Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.
Cambridge International is publishing the mark schemes for the May/June 2019 series for most Cambridge IGCSE ${ }^{\text {TM }}$, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

## GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.


## GENERIC MARKING PRINCIPLE 2 :

Marks awarded are always whole marks (not half marks, or other fractions).

## GENERIC MARKING PRINCIPLE 3:

Marks must be awarded positively:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.


## GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

## GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

## Mark Scheme Notes

Marks are of the following three types:
M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier $M$ or $B$ (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0. $B 2 / 1 / 0$ means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking $g$ equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:
AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO Correct Working Only - often written by a 'fortuitous' answer
ISW Ignore Subsequent Working
SOI Seen or implied
SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

## Penalties

MR -1 A penalty of MR-1 is deducted from $A$ or $B$ marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR - 2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA -1 This is deducted from $A$ or $B$ marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 1(i) | $\Sigma(t-120)=-25+6-3+15+0+5-6-1+16=7$ | M1 | Attempt to sum both $(t-120)$ and $(t-120)^{2}$ Correct ans using $\Sigma t-9 \times 120$ and $\Sigma(t-120)^{2} \mathrm{M} 1 \mathrm{~A} 1$ |
|  | $\begin{aligned} & \Sigma(t-120)^{2}=25^{2}+6^{2}+3^{2}+15^{2}+0^{2}+5^{2}+6^{2}+1^{2}+16^{2} \\ & =1213 \end{aligned}$ | A1 | Both correct, www, SC correct ans no working B1B1 |
|  |  | 2 |  |
| 1(ii) | $\operatorname{Var}=\frac{\sum(t-120)^{2}}{9}-\left(\frac{\sum(t-120)}{9}\right)^{2}=\frac{t h e i r 1213}{9}-\left(\frac{\text { their } 7}{9}\right)^{2}$ | M1 | Using two coded values in correct formula including finding $\Sigma t$ from 7 etc |
|  | $=134(2)$ | A1 | Correct answer <br> SC if correct variance obtained by another method from raw data give SCB1 |
|  |  | 2 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 2 | Jameal: P (plum) $=\frac{5}{8}$ Rosa: P (plum) $=\frac{x}{}$ | M1 | Their 2 probabilities for P (plum) multiplied and equated to $1 / 4$ |
|  | $\frac{5}{8} \times \frac{x}{x+6}=\frac{1}{4}$ | A1 | Correct equation oe |
|  | $(x=) 4$ | A1 | SC correct answer with no appropriate equations i.e. common sense B1 |
|  |  | 3 |  |


| Question | Answer | Marks | Guidance |
| :---: | :--- | ---: | :--- |
| 3 | $\mathrm{P}(\mathrm{X})=\frac{3}{36}\left(\frac{1}{12} o e\right)$ | B1 |  |
|  | $\mathrm{P}(\mathrm{Y})=\frac{12}{36}\left(\frac{1}{3} o e\right)$ | B1 |  |
|  | $\mathrm{P}(\mathrm{X} \cap \mathrm{Y})=\frac{1}{36}$ | M1 | Independent method to find $\mathrm{P}(X \cap Y)$ without multiplication, either <br> stated or by listing or circling numbers on a probability space <br> diagram. OR condititional prob with a single fraction numerator |
|  | $\mathrm{P}(\mathrm{X}) \times \mathrm{P}(\mathrm{Y})=\mathrm{P}(\mathrm{X} \cap \mathrm{Y})$, independent | A1 | Numerical comparison and conclusion, www |
|  |  | $\mathbf{4}$ |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 4 | Median Maths $=40$ | M1 | Indication of finding medians, such as mark on graph or reference marks to 700 pupils, condone poor terminology such as 'mean' |
|  | Median English $=55$ | A1 | Both values correct, condone $54<$ English $<56$ but 54 , 56 get A0 |
|  | Median of English is larger than median of Maths | B1 | Correct statement, median must be referenced within answer. No credit if statement references 'means' |
|  | Range Maths is 100 or IQ range Maths $=80-12=68$ | M1 | Evidence of finding either both ranges or both IQ ranges i.e. see a minus |
|  | Range English is 60 or IQ range English $=62-42=20$ | A1 | Both ranges or IQR correct |
|  | Maths marks have more spread then English marks | B1 | Correct conclusion. Accept standard deviation but must see some figures |
|  |  | 6 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 5(i) | $(\mathrm{P}>12)=\mathrm{P}(13,14,15)$ | M1 | Binomial term of form ${ }^{15} \mathrm{C}_{x} p^{x}(1-p)^{15-x} 0<p<1$ any $p, x \neq 15,0$ |
|  | $={ }^{15} \mathrm{C}_{13}(0.65)^{13}(0.35){ }^{2}+{ }^{15} \mathrm{C}_{14}(0.65){ }^{14}(0.35){ }^{1}+(0.65)^{15}$ | A1 | Correct unsimplified answer |
|  | $=0.0617$ | A1 | SC if use np and npq with justification give $(12.5-9.75) / \sqrt{3.41} \mathrm{M} 1$ $1-\mathrm{F}(1.489) \mathrm{A} 10.0681 \mathrm{~A} 0$ |
|  |  | 3 |  |
| 5(ii) | $\begin{aligned} & \text { mean }=250 \times 0.65=162.5 \\ & \text { variance }=250 \times 0.65 \times 0.35=56.875 \end{aligned}$ | B1 | Correct unsimplified $n p$ and $n p q$ |
|  | $\mathrm{P}(<179)=\mathrm{P}\left(\mathrm{z}<\frac{178.5-162.5}{\sqrt{56.875}}\right)=\mathrm{P}(\mathrm{z}<2.122)$ | M1 | Substituting their $\mu$ and $\sigma$ (condone $\sigma^{2}$ ) into the Standardisation Formula with a numerical value for ' 178.5 '. Continuity correct not required for this M1. Condone $\pm$ standardisation formula |
|  | Using continuity correction 178.5 or 179.5 | M1 |  |
|  | $=0.983$ | A1 | Correct final answer |
|  |  | 4 |  |


| Question | Answer | Marks | Guidance |
| :---: | :--- | ---: | ---: |
| $6(\mathrm{i})$ | $\mathrm{P}($ loses $\$ 1)=\mathrm{P}(\mathrm{F}$ and F$)=0.8 \times 0.8$ | M1 | $0.8 \times 0.8$ or $(1-0.2)(1-0.2)$ or $\mathrm{P}(\mathrm{F}) \times \mathrm{P}(\mathrm{F})$ or $\mathrm{P}(\mathrm{F})+\mathrm{P}(\mathrm{F})$ seen or <br> implied |
|  | $=0.64 \mathrm{AG}$ | $\mathbf{A 1}$ | Must see probabilities multiplied together with final answer and a <br> clear probability statement or implied by labelled tree diagram |
|  |  | $\mathbf{2}$ |  |


| Question | Answer |  |  |  | Marks | Guidance |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6(ii) | Amount gained (\$) | -1 | 0.50 | 2 | B1 | -1 linked with 0.64 in table |
|  |  |  |  |  | B1 | 0.5 seen in table |
|  | Prob |  | 0.16 | 0.2 | B1 | 0.16 seen in table linked to their 0.5 |
|  |  |  |  |  | B1 | FT $\mathrm{P}(2.00$ gained $)=0.36-\mathrm{P}(0.50$ gained $)$ or correct, and all amount gained linked correctly in table |
|  |  |  |  |  | 4 |  |
| 6(iii) | $\begin{aligned} & \mathrm{E}(\text { winnings })=-1 \times 0.64+0.5 \times 0.16+2 \times 0.2 \\ & \quad=-(\$) 0.16,-16 \text { cents } \end{aligned}$ |  |  |  | B1 | FT Accept (\$)0.16 or 16 cents loss. FT unsimplified E(winnings) from their table provided $\Sigma p=1$ |
|  |  |  |  |  | 1 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 7(i) | $\mathrm{P}(<700)=\mathrm{P}\left(z<\frac{700-830}{120}\right)=\mathrm{P}(\mathrm{z}<-1.083)$ | M1 | Using $\pm$ standardisation formula, no continuity correction, not $\sigma^{2}$ or $\sqrt{ } \sigma$ |
|  | $=1-0.8606$ | M1 | Appropriate area $\Phi$ from standardisation formula $\mathrm{P}(\mathrm{z}<\ldots$.$) in final$ probability solution, ( $<0.5$ if $z$ is $-\mathrm{ve},>0.5$ if $z$ is +ve ) |
|  | $=0.1394$ | A1 | Correct final probability rounding to 0.139 |
|  | Expected number of female adults $=430 \times$ their 0.1394 $=59.9$ So 59 or 60 | B1 | FT their 3 or 4 SF probability, rounded or truncated to integer |
|  |  | 4 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 7(ii) | $\mathrm{P}($ giraffe $<830+w)=95 \%$ so $z=1.645$ | B1 | $\pm 1.645$ seen (critical value) |
|  | $\frac{(830+w)-830}{120}=\frac{w}{120}=1.645$ | M1 | An equation using the standardisation formula with a $z$-value (not $1-z$ ), condone $\sigma^{2}$ or $\sqrt{ } \sigma$ not $0.8519,0.8289$ |
|  | $w=197$ | A1 | Correct answer |
|  |  | 3 |  |
| 7(iii) | $\mathrm{P}($ male $>950)=0.834$, so $z=-0.97$ | B1 | $\pm 0.97$ seen |
|  | $\frac{950-1190}{\sigma}=-0.97$ | M1 | Using $\pm$ standardisation formula, condone continuity correction, $\sigma^{2}$ or $\sqrt{ } \sigma$, condone equating with non $z$-value not $0.834,0.166$ |
|  | $\sigma=247$ | A1 | Condone $-\sigma=-247$. www. |
|  |  | 3 |  |


| Question | Answer | Marks | Guidance |
| :---: | :--- | ---: | ---: |
| $8(\mathrm{i})$ | $\left({ }^{9} \mathrm{C}_{4}=\right) 126$ | $\mathbf{B 1}$ |  |
|  |  | $\mathbf{1}$ |  |
| $8(\mathrm{ii})$ | ${ }^{7} \mathrm{C}_{2}$ | $\mathbf{B 1}$ | ${ }^{7} \mathrm{C}_{\mathrm{x}}$ or ${ }^{\mathrm{y}} \mathrm{C}_{2}$ (implied by correct answer) or ${ }^{7} \mathrm{P}_{\mathrm{x}}$ or ${ }^{7} \mathrm{P}_{\mathrm{y}}$, seen alone |
|  | $=21$ | $\mathbf{B 1}$ | correct answer |
|  |  | $\mathbf{2}$ |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 8(iii) | ${ }_{-} \mathrm{C}_{1}\left(\mathrm{~B}_{1} \mathrm{~B}_{2} \mathrm{~B}_{3}\right) \mathrm{C}_{2}{ }_{-} \mathrm{C}_{3}{ }_{-} \mathrm{C}_{4}{ }_{-} \mathrm{C}_{5}{ }_{-} \mathrm{C}_{6}$ | B1 | 3 ! or 6 ! seen alone or multiplied by $\mathrm{k}>1$ need not be an integer |
|  | $3!\times 6!\times 7$ | B1 | 3 ! and 6 ! seen multiplied by $\mathrm{k}>1$, integer, no division |
|  | $=30240$ | B1 | Exact value |
|  | Alternative method for question 8(iii) |  |  |
|  | $\mathrm{C}_{1}\left(\mathrm{~B}_{1} \mathrm{~B}_{2} \mathrm{~B}_{3}\right) \mathrm{C}_{2} \mathrm{C}_{3} \mathrm{C}_{4} \mathrm{C}_{5} \mathrm{C}_{6}$ | B1 | 3 ! or 7 ! seen alone or multiplied by $\mathrm{k}>1$ need not be an integer |
|  | $3!\times 7$ ! | B1 | 3 ! and 7 ! seen multiplied by $\mathrm{k}>$ or $=1$, no division |
|  | $=30240$ | B1 | Exact value |
|  |  | 3 |  |
| 8(iv) | $\mathrm{C}_{1}{ }_{-} \mathrm{C}_{2} \__{-} \mathrm{C}_{3} \mathrm{C}_{4}{ }_{-} \mathrm{C}_{5} \mathrm{C}_{6}$ | B1 | 6 ! or 4 ! X 6P2 seen alone or multiplied by $\mathrm{k}>1$, no division (arrangements of cars) |
|  | $6!\times 5 \mathrm{P} 3$ or $6!\times 5 \times 4 \times 3$ or $6!\times 3!\times 10$ | B1 | Multiply by 5P3 oe i.e. putting Bs in between 4 of the Cs OR multiply by 3 ! $\times \mathrm{n}$ where $\mathrm{n}=7,8,9,10$ (number of options) |
|  | $=43200$ | B1 | Correct answer |
|  |  | 3 |  |

